COMPARISON OF THE METHODS USAGE IN EIT IMAGE RECONSTRUCT

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Abstract: The article compares the most widely used regularization methods used in electrical impedance tomography image reconstruction. The level set method is described. Origin use of this method is in image processing for image segmentation. Implementation of the level set method into algorithm of EIT image reconstruction can give as results with very good resolution. In the article results from three methods are presented a compared.

Keywords: EIT, FEM, level set method, tikhonov regularization method, total variation method

1. INTRODUTION

The electrical impedance tomography (EIT) is a widely investigated problem with many applications in industry and biological science. EIT is non-invasive method, which is a big advantage for medical applications. Can by also used for continuous monitoring of human body. This method is used as a complementary diagnostic method for investigation of the breast cancer, brain monitoring and heard function diagnosis. As well as mentioned fields of use the detection of pulmonary emboli and circulation of blood is also possible. Application of the EIT is in different lines of industry like a chemical industry for mixing process monitoring, separation, monitoring of chemical reaction and recognition of concentration alloys. In materiology it is used for recognition of non-homogenous areas, defects and corrosion in materials and products and for location of conductivity deposits and porosity information in geological industry.

2. ELECTRICAL IMPEDANCE TOMOGRAPHY

Passive electrical properties are utilized in EIT. Basic principle of EIT is: voltage electrodes are connected at the edge of investigated subject. Usual number of electrodes is 16 or 32. Voltage on these electrodes is measured while the low frequency current source is connected to subject. Placement of the voltages electrodes and the current source is in figure 1. We can obtain conductivity distribution inside subject by solving inverse task. Vector of measured voltages is used for reconstruction. It is well known that inverse task is non-stable and ill-posed problem. The regularization methods are used for stability of inverse task. Numerical methods can be used to obtained voltage vector for testing of reconstruction methods in EIT. Further, for simplicity we will consider only on the conductivity σ. The scalar potential U can be therefore introduced, and so the resulting field is conservative and the continuity equation for the current density can be expressed by the potential U

$$\mathrm{div}(\sigma\,\mathrm{grad}U) = 0\,. \tag{1}$$

Equation (1) together with the modified complete electrode model equations (described in [1,2,3]) are discretized by the finite element method in the usual way. By the finite element method can by calculated approximate values of electrode voltages for the approximate element conductivity vector σ (NE x 1), NE is the number of finite elements. Furthermore, we assume the constant approximation of the conductivity σ on each of all elements.

3. REGULARIZATION METHODS AND LEVEL SET METHOD THEORY

There are used deterministic and stochastic for EIT reconstruction. Deterministic approach for minimizing suitable objective function use Newton iteration method, leas square method, steepest descent method and conjugate gradient method. Stochastic approaches use probability theory, heuristic methods or genetic algorithms. Both approaches have advantages and disadvantages. Deterministic methods are use in this work. There is minimizing suitable objective function by least square method. There is add regularization term to objective function to obtain stable solution. Tikhonov regularization method (TR) and Total variation method (TV) are most widely used in EIT image reconstruction and used in this work.

3.1. TIKHONOV REGULARIZATION METHOD

Suitable objective function for Tikhonov regularization method is

$$\Psi\left(\sigma\right)=\frac{1}{2}\sum\left\|\boldsymbol{U}_{\mathrm{M}}-\boldsymbol{U}_{\mathrm{FEM}}\right\|^{2}+\alpha\left\|\mathbf{R}\sigma\right\|^{2}, \tag{2}$$

where σ is vector of unknown conductivity, U_M is vector of voltage on electrodes on the edge of the subject, $U_{FEM}(\sigma)$ is vector calculated on the edge of the subject, α is regularization parameter and **R** is regularization matrix which describe connection between elements.

Regularization matrix is square matrix which describes connection between conductivity distribution and mesh elements. Creation can be described: at i-row and j-column is -1 for contiguous elements and count of contiguous elements at main diagonal.

Gauss-Newton recursion can be writing in form

$$\boldsymbol{\sigma}_{i+1}=\boldsymbol{\sigma}_{i}+(\mathbf{J}_{i}^{\mathrm{T}}\mathbf{J}_{\mathrm{i}}+\alpha\mathbf{R}^{\mathrm{T}}\mathbf{R})^{-1}\left(\mathbf{J}_{i}^{\mathrm{T}}\left(U_{\mathrm{M}}-U_{\mathrm{FEM}}\left(\sigma_{i}\right)\right)-\alpha\mathbf{R}^{\mathrm{T}}\mathbf{R}\boldsymbol{\sigma}_{i}\right). \tag{3}$$

Jacobian describe sensitivity of electrode potentials on change of element conductivity

$$\mathbf{J}=\frac{\partial U_{i}}{\partial\sigma}\begin{pmatrix}\frac{\partial U_{1\mathrm{i}}}{\partial\sigma_{1}} & \cdots & \frac{\partial U_{1\mathrm{i}}}{\partial\sigma_{NE}}\\ \vdots & \ddots & \vdots\\ \frac{\partial U_{L\mathrm{i}}}{\partial\sigma_{1}} & \cdots & \frac{\partial U_{L\mathrm{i}}}{\partial\sigma_{NE}}\end{pmatrix}. \tag{4}$$

3.2. TOTAL VARIATION METHOD

Suitable objective function for Tikhonov regularization method is

$$\Psi(\sigma)=\frac{1}{2}\sum(\boldsymbol{U}_{\mathrm{M}}-\boldsymbol{U}_{\mathrm{FEM}}(\sigma))^{2}+\alpha\mathrm{TV}_{\beta}, \tag{5}$$

where σ is vector of unknown conductivity, U_M is vector of voltages on electrodes on subject boundary, $U_{FEM}(\sigma)$ is vector calculated on subject boundary by, α is regularization parameter and

$$\mathrm{TV}_{\beta}=\sum_{NE}\int\left|\mathrm{grad}\sigma\right|d\Omega=\sum\sqrt{\left\|\mathbf{R}\sigma\right\|+\beta}, \tag{6}$$

where **R** is regularization matrix which describe connection between elements and β is small positive parameter.

There were created several versions of this method. One of them is PD-IPM (Primal-Dual Interior-Point-Method). This version has dimension of regularization matrix (count of mesh edges x count of elements). Base of PD-IPM algorithm is interpolation between lagged diffusivity method a Newton's method [1,2].

3.3. LEVEL SET METHOD

This method is used in image processing for image segmentation. LS method is used in algorithm for EIT image reconstruction and with some regularization method give as very good results. Basic principle of this method is in application of function $\phi(\boldsymbol{r};t)$ on the image, where $\boldsymbol{r}$ pointer in space and t is time step. Initialization of the function $\phi(\boldsymbol{r};t)$ is in time t = 0. It is traced evolution of this function. Function is updated in small time intervals. Level set curve is defined as all points with zero value of function $\phi(\boldsymbol{r};t_0)$ [4,5].

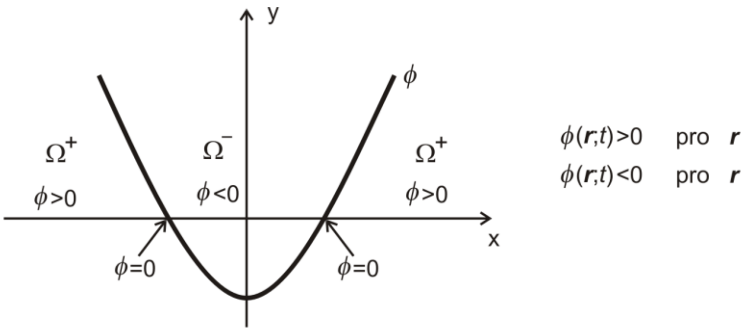

Figure 1: Evolution level set function in time t

4. NUMERICAL MODELS DESCTRIBTION

An arrangement of task is in Figure 1. Numerical model for comparison of methods in EIT reconstruction consist of 300 elements and 167 nodes. FEM grid is shown in figure 2. Current excitation with trigonometric distribution has been used in the model. The 16 voltage electrodes have been used for measurement of voltage at boundary of investigated object. There were created four numerical models for comparison of results obtained by described methods for EIT image reconstruction. There are two values of conductivity in the tested models. Homogenous region has conductivity σ = 300 S/m and defect (non-homogenous region) has conductivity σ = 50 S/m. Both regularization methods were used without changes. New approach was used for other numerical models. For solving of this task was used combination of the regularization methods and Level Set method for searching of the non-homogenous regions. The principle of new approach is approximating of the non-homogenous regions by one regularization method. The level set method was applied to identifying of the location of the non-homogenous regions. There was applied one of the regularization method again on the segmented non-homogenous region by the level set method and calculated final conductivity distribution [6].

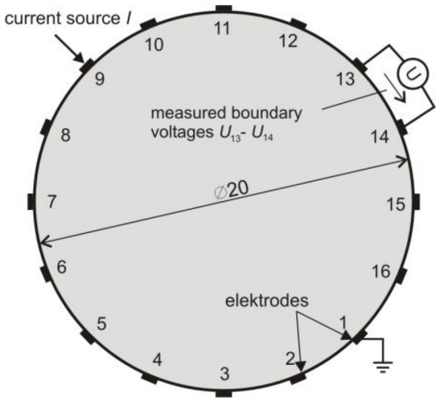

Figure 2: Arrangement of task

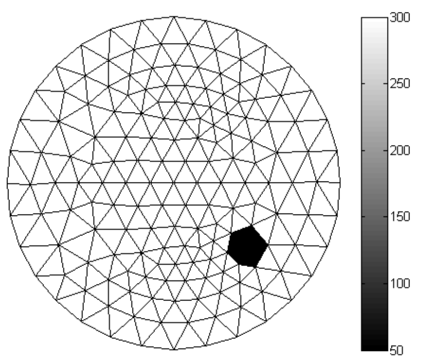

Figure 3: Mesh grid, original conductivity distribution

5. RESULTS

The obtained results from tested methods are presented in this part. Origin conductivity distribution is in figure 3. Final conductivity distribution obtained by TR only is shown in figure 4. How we can see that non-homogenous region is recognized but final value of the conductivities on elements is of non-homogenous region is variety of original value. Final conductivity distribution obtained by TV only is shown in figure 5. This method recognized non-homogenous region like TR. Final values of conductivities on elements are worse than conductivities obtained by TR. Results obtained in steps of algorithm which combination TR-LS-TR are in figures 6. and 7. Results after usage TR for find non-homogenous area is in figure 6. This non-homogenous region can by found approximately (smaller iteration count). Non-homogenous region and surround selected by LS in shown in figure 7. The TR was used for found final conductivity value. Original and final conductivity distribution is equal with figure 3.

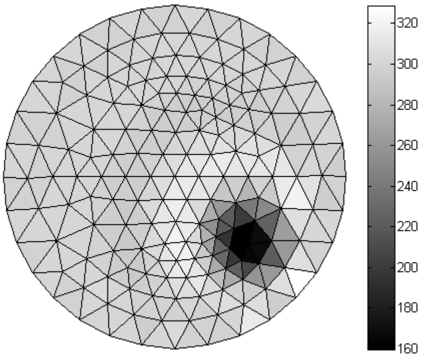

Figure 4: TR only - conductivity distribution

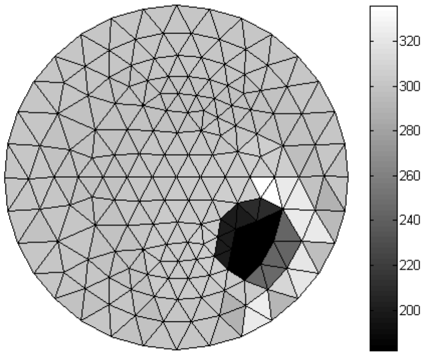

Figure 5: TV only - conductivity distribution

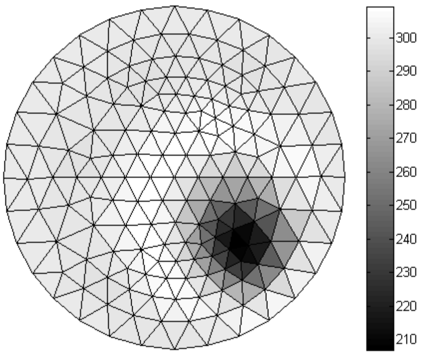

Figure 6: Found region by TR

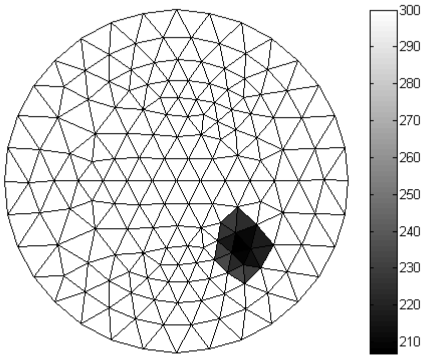

Figure 7: LS level

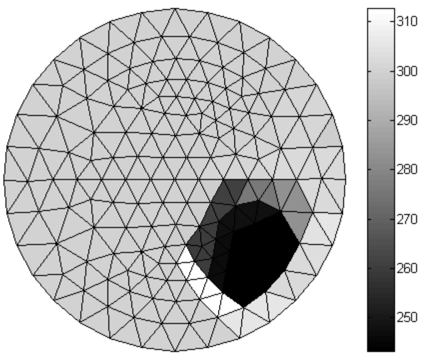

Figure 8: Found region by TV

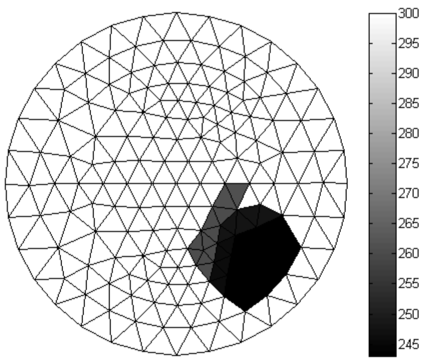

Figure 9: LS level

Results from combination TV-LS-TV are shown in figures 8. and 9. Meaning of this figures are same like figure 6. and 7. Final conductivity distribution is equal as original.

Method	Time [s]	Iterations
TR	0.5098	327
TV	0.4329	131
TR-LS-TR	0,2596	86
TV-LS-TV	0,2751	47

Table 1: Time and iterations count for compared methods

Time consuming of compared methods is in table 1. We can see that TR only with 327 iterations is less time consuming than TV only with 131 iterations. If the new approach was used with combination TR-LS-TR solution time was half. For combination TV-LS-TV is time one-third.

6. CONCLUSION

We can see from obtained results that both regularization methods identify non-homogenous region. Final conductivity distribution is not equal with origin distribution. Non-homogenous region recognition by TR is better than by TV but TR needs more iterations for solution. This property of the regularization methods can be utilized for defects surround finding and consequently use the LS for selecting the defect. For finding of the final conductivity distribution is again applied regularization method. New algorithm used in EIT image reconstruction is very effective and less time consumed than use of the regularization method only.

Measuring workplace is already prepared at this time. The sixteen electrodes will by placed on the edge of the specimen. Excitation current will by 1 A. The voltage will by measured on adjoining electrodes. This measured values will be used for internal conductivity reconstruction. Experiment and simulations results will by compared.

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REFERENCES

[1] Cheney, M., Isaacson, D., Newell, J., C., Electrical impedance tomography. SIAM Rev., vol. 41, no. 1, 1999, p. 85-101.

[2] Borsic, A. Regularization methods for imaging from electrical measurement. PhD. Thesis, Oxford Brookes University, 2002.

[3] CHENG, K. S., ISAACSON, D., NEWELL, J. C., GISSER D. G. Electrode models for electric current computed tomography. IEEE Trans Biomed Eng, p. 918-924, 1989.

[4] Sethiah, J., A. Level set methods and fast marching methods. Cambridge, Cambridge University Press, 1999.

[5] Osher, S., Fedkiw, R. Level set methods and dynamic implicit surfaces, Springer-Verlag, New York, 2002.

[6] Kříž, T.; Dědková J. A New Algorithm for Electrical Impedance Tomography Inverse Problem. In PIERS Proceedings. Progress In Electromagnetics. Cambridge: The Electromagnetic Academy, 2009. s. 132-136. ISBN: 978-1-934142-08- 0.